Tuesday, FEBRUARY 12, 2002

Contest A

The MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions
Presented by the Akamai Foundation

AMC 10

3rd Annual American Mathematics Contest 10

1. DO NOT OPEN THIS BOOKLET UNTIL TOLD TO DO SO BY YOUR PROCTOR.
2. This is a twenty-five question, multiple choice test. Each question is followed by answers marked A,B,C,D and E. Only one of these is correct.
3. The answers to the problems are to be marked on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 2.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, erasers and calculators that are accepted for use on the SAT. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** working time to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score in the top 1% on this AMC 10 will be invited to take the 20th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 26, 2002 or on Tuesday, April 9, 2002. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

1. The ratio $\frac{10^{2000}+10^{2002}}{10^{2001}+10^{2001}}$ is closest to which of the following numbers?

   (A) 0.1 **(B)** 0.2 **(C)** 1 **(D)** 5 **(E)** 10

2. For the nonzero numbers a, b, and c, define

$$(a, b, c) = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}.$$

   Find $(2, 12, 9)$.

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

3. According to the standard convention for exponentiation,

$$2^{2^{2^{2}}} = 2^{\left(2^{\left(2^{2}\right)}\right)} = 2^{16} = 65,536.$$

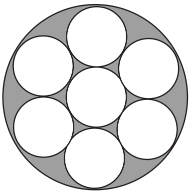
   If the order in which the exponentiations are performed is changed, how many other values are possible?

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. For how many positive integers m does there exist at least one positive integer n such that $m \cdot n \leq m + n$?

   (A) 4 **(B)** 6 **(C)** 9 **(D)** 12 **(E)** infinitely many

5. Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.



   (A) π **(B)** 1.5π **(C)** 2π **(D)** 3π **(E)** 3.5π

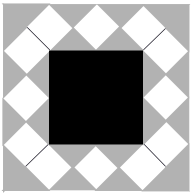
6. Cindy was asked by her teacher to subtract 3 from a certain number and then divide the result by 9. Instead, she subtracted 9 and then divided the result by 3, giving an answer of 43. What would her answer have been had she worked the problem correctly?

(A) 15 **(B)** 34 **(C)** 43 **(D)** 51 **(E)** 138

7. If an arc of 45° on circle A has the same length as an arc of 30° on circle B, then the ratio of the area of circle A to the area of circle B is

(A) $\frac{4}{9}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{5}{6}$ **(D)** $\frac{3}{2}$ **(E)** $\frac{9}{4}$

8. Betsy designed a flag using blue triangles (■), small white squares (□), and a red center square(■), as shown. Let B be the total area of the blue triangles, W the total area of the white squares, and R the area of the red square. Which of the following is correct?



(A) $B = W$ **(B)** $W = R$ **(C)** $B = R$ **(D)** $3B = 2R$ **(E)** $2R = W$

9. Suppose A, B, and C are three numbers for which $1001C - 2002A = 4004$ and $1001B + 3003A = 5005$. The average of the three numbers A, B, and C is

(A) 1 **(B)** 3 **(C)** 6 **(D)** 9 **(E)** not uniquely determined

10. Compute the sum of all the roots of $(2x + 3)(x - 4) + (2x + 3)(x - 6) = 0$.

(A) 7/2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 13

11. Jamal wants to store 30 computer files on floppy disks, each of which has a capacity of 1.44 megabytes (mb). Three of his files require 0.8 mb of memory each, 12 more require 0.7 mb each, and the remaining 15 require 0.4 mb each. No file can be split between floppy disks. What is the minimal number of floppy disks that will hold all the files?

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

12. Mr. Earl E. Bird leaves his house for work at exactly 8:00 A.M. every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

(A) 45 **(B)** 48 **(C)** 50 **(D)** 55 **(E)** 58

13. The sides of a triangle have lengths of 15, 20, and 25. Find the length of the shortest altitude.

(A) 6 **(B)** 12 **(C)** 12.5 **(D)** 13 **(E)** 15

14. Both roots of the quadratic equation $x^2 - 63x + k = 0$ are prime numbers. The number of possible values of k is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** more than four

15. The digits 1, 2, 3, 4, 5, 6, 7, and 9 are used to form four two-digit prime numbers, with each digit used exactly once. What is the sum of these four primes?

(A) 150 **(B)** 160 **(C)** 170 **(D)** 180 **(E)** 190

16. If $a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5$, then $a + b + c + d$ is

(A) -5 **(B)** $-10/3$ **(C)** $-7/3$ **(D)** $5/3$ **(E)** 5

17. Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

(A) $1/4$ **(B)** $1/3$ **(C)** $3/8$ **(D)** $2/5$ **(E)** $1/2$

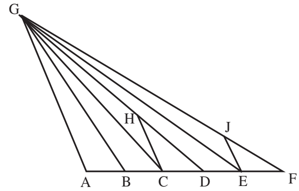
18. A $3 \times 3 \times 3$ cube is formed by gluing together 27 standard cubical dice. (On a standard die, the sum of the numbers on any pair of opposite faces is 7.) The smallest possible sum of all the numbers showing on the surface of the $3 \times 3 \times 3$ cube is

(A) 60 **(B)** 72 **(C)** 84 **(D)** 90 **(E)** 96

19. Spot's doghouse has a regular hexagonal base that measures one yard on each side. He is tethered to a vertex with a two-yard rope. What is the area, in square yards, of the region outside the doghouse that Spot can reach?

(A) $\frac{2}{3}\pi$ **(B)** 2π **(C)** $\frac{5}{2}\pi$ **(D)** $\frac{8}{3}\pi$ **(E)** 3π

20. Points A, B, C, D, E, and F lie, in that order, on $\overline{AF}$, dividing it into five segments, each of length 1. Point G is not on line AF. Point H lies on $\overline{GD}$, and point J lies on $\overline{GF}$. The line segments $\overline{HC}$, $\overline{JE}$, and $\overline{AG}$ are parallel. Find HC/JE.



(A) 5/4 **(B)** 4/3 **(C)** 3/2 **(D)** 5/3 **(E)** 2

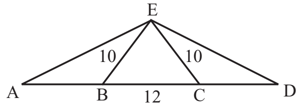
21. The mean, median, unique mode, and range of a collection of eight integers are all equal to 8. The largest integer that can be an element of this collection is

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

22. A set of tiles numbered 1 through 100 is modified repeatedly by the following operation: remove all tiles numbered with a perfect square, and renumber the remaining tiles consecutively starting with 1. How many times must the operation be performed to reduce the number of tiles in the set to one?

(A) 10 **(B)** 11 **(C)** 18 **(D)** 19 **(E)** 20

23. Points A, B, C, and D lie on a line, in that order, with $AB = CD$ and $BC = 12$. Point E is not on the line, and $BE = CE = 10$. The perimeter of $\triangle AED$ is twice the perimeter of $\triangle BEC$. Find AB.



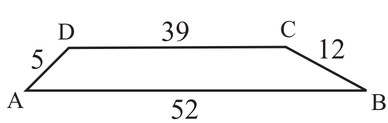
(A) 15/2 **(B)** 8 **(C)** 17/2 **(D)** 9 **(E)** 19/2

24. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, \ldots, 10\}$. The probability that Sergio's number is larger than the sum of the two numbers chosen by Tina is

(A) 2/5 **(B)** 9/20 **(C)** 1/2 **(D)** 11/20 **(E)** 24/25

25. In trapezoid $ABCD$ with bases $\overline{AB}$ and $\overline{CD}$, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. The area of $ABCD$ is

(A) 182 **(B)** 195 **(C)** 210 **(D)** 234 **(E)** 260



WRITE TO US!

*Correspondence about the problems and solutions for this **AMC 10** should be addressed to:*

Prof. Douglas Faires, Department of Mathematics
Youngstown State University, Youngstown, OH 44555-0001
Phone:330-742-1805; Fax: 330-742-3170; email: faires@math.ysu.edu

Orders for any of the publications listed below should be addressed to:

Titu Andreescu, Director
American Mathematics Competitions
University of Nebraska, P.O. Box 81606
Lincoln, NE 68501-1606
Phone: 402-472-2257; Fax: 402-472-6087; email: titu@amc.unl.edu;

2002 AIME

The AIME will be held on Tuesday, March 26, 2002 with the alternate on April 9,2002. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score in the top 1% of this AMC 10 or receive a score of 100 or above on the AMC 12. Alternately, you must be in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the USA Mathematical Olympiad (USAMO) on Thursday through Sunday, May 9-12, 2002 in Cambridge, MA. The best way to prepare for the AIME and USAMO is to study previous years of these exams. Copies may be ordered as indicated below.

PUBLICATIONS

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Examinations: Each price is for one copy of an exam and its solutions for one year. Specify the years you want and how many copies of each. All prices effective to September 1, 2002.

- AMC 10 2000-2002/AHSME (AMC 12) 1989-2002, $1 per copy per year.
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2002

AMC 10 - Contest A

DO NOT OPEN UNTIL

Tuesday, FEBRUARY 12, 2002

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. **PLEASE READ THE MANUAL BEFORE FEBRUARY 12.** Nothing is needed from inside this package until February 12.
2. Your PRINCIPAL or VICE PRINCIPAL must sign the Certification Form A found in the Teachers' Manual.
3. The Answer Forms must be mailed by First Class mail to the AMC Director, Titu Andreescu, no later than 24 hours following the examination.
4. <u>Please Note:</u> All Problems and Solutions are copyrighted; it is illegal to make copies or transmit them on the internet or world wide web without permission.
5. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Duplication at any time via copier, telephone, eMail, World Wide Web or media of any type is a violation of the copyright law.*

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