



## AP<sup>®</sup> Calculus AB 2003 Scoring Guidelines

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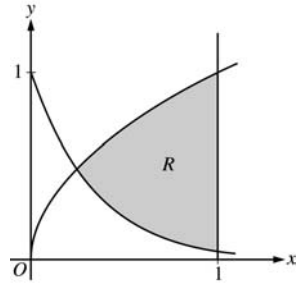
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**Question 1**

Let  $R$  be the shaded region bounded by the graphs of  $y = \sqrt{x}$  and  $y = e^{-3x}$  and the vertical line  $x = 1$ , as shown in the figure above.

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is revolved about the horizontal line  $y = 1$ .
- (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a rectangle whose height is 5 times the length of its base in region  $R$ . Find the volume of this solid.



Point of intersection

$$e^{-3x} = \sqrt{x} \text{ at } (T, S) = (0.238734, 0.488604)$$

$$\begin{aligned} \text{(a) Area} &= \int_T^1 (\sqrt{x} - e^{-3x}) dx \\ &= 0.442 \text{ or } 0.443 \end{aligned}$$

$$\begin{aligned} \text{(b) Volume} &= \pi \int_T^1 \left( (1 - e^{-3x})^2 - (1 - \sqrt{x})^2 \right) dx \\ &= 0.453\pi \text{ or } 1.423 \text{ or } 1.424 \end{aligned}$$

$$\begin{aligned} \text{(c) Length} &= \sqrt{x} - e^{-3x} \\ \text{Height} &= 5(\sqrt{x} - e^{-3x}) \end{aligned}$$

$$\text{Volume} = \int_T^1 5(\sqrt{x} - e^{-3x})^2 dx = 1.554$$

1: Correct limits in an integral in

(a), (b), or (c)

2:  $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{answer} \end{array} \right.$

3:  $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ reversal} \\ < -1 > \text{ error with constant} \\ < -1 > \text{ omits 1 in one radius} \\ < -2 > \text{ other errors} \\ 1 : \text{answer} \end{array} \right.$

3:  $\left\{ \begin{array}{l} 2 : \text{integrand} \\ < -1 > \text{ incorrect but has} \\ & \sqrt{x} - e^{-3x} \\ & \text{as a factor} \\ 1 : \text{answer} \end{array} \right.$

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**Question 2**

A particle moves along the  $x$ -axis so that its velocity at time  $t$  is given by

$$v(t) = -(t + 1)\sin\left(\frac{t^2}{2}\right).$$

At time  $t = 0$ , the particle is at position  $x = 1$ .

- (a) Find the acceleration of the particle at time  $t = 2$ . Is the speed of the particle increasing at  $t = 2$ ? Why or why not?
- (b) Find all times  $t$  in the open interval  $0 < t < 3$  when the particle changes direction. Justify your answer.
- (c) Find the total distance traveled by the particle from time  $t = 0$  until time  $t = 3$ .
- (d) During the time interval  $0 \leq t \leq 3$ , what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

- (a)  $a(2) = v'(2) = 1.587$  or  $1.588$   
 $v(2) = -3\sin(2) < 0$   
 Speed is decreasing since  $a(2) > 0$  and  $v(2) < 0$ .

$$2 : \begin{cases} 1 : a(2) \\ 1 : \text{speed decreasing} \\ \text{with reason} \end{cases}$$

- (b)  $v(t) = 0$  when  $\frac{t^2}{2} = \pi$   
 $t = \sqrt{2\pi}$  or  $2.506$  or  $2.507$   
 Since  $v(t) < 0$  for  $0 < t < \sqrt{2\pi}$  and  $v(t) > 0$  for  $\sqrt{2\pi} < t < 3$ , the particle changes directions at  $t = \sqrt{2\pi}$ .

$$2 : \begin{cases} 1 : t = \sqrt{2\pi} \text{ only} \\ 1 : \text{justification} \end{cases}$$

- (c) Distance =  $\int_0^3 |v(t)| dt = 4.333$  or  $4.334$

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

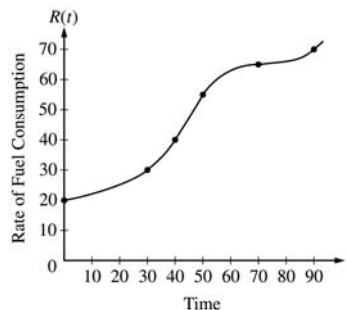
- (d)  $\int_0^{\sqrt{2\pi}} v(t) dt = -3.265$   
 $x(\sqrt{2\pi}) = x(0) + \int_0^{\sqrt{2\pi}} v(t) dt = -2.265$   
 Since the total distance from  $t = 0$  to  $t = 3$  is  $4.334$ , the particle is still to the left of the origin at  $t = 3$ . Hence the greatest distance from the origin is  $2.265$ .

$$2 : \begin{cases} 1 : \pm \text{ (distance particle travels} \\ \text{while velocity is negative)} \\ 1 : \text{answer} \end{cases}$$

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**Question 3**

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above.



$t$ (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

- (a) Use data from the table to find an approximation for  $R'(45)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time  $t = 45$  minutes. What is the value of  $R''(45)$ ? Explain your reasoning.
- (c) Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain your reasoning.
- (d) For  $0 < b \leq 90$  minutes, explain the meaning of  $\int_0^b R(t) dt$  in terms of fuel consumption for the plane. Explain the meaning of  $\frac{1}{b} \int_0^b R(t) dt$  in terms of fuel consumption for the plane. Indicate units of measure in both answers.

(a) 
$$R'(45) \approx \frac{R(50) - R(40)}{50 - 40} = \frac{55 - 40}{10}$$

$$= 1.5 \text{ gal/min}^2$$

(b)  $R''(45) = 0$  since  $R'(t)$  has a maximum at  $t = 45$ .

(c) 
$$\int_0^{90} R(t) dt \approx (30)(20) + (10)(30) + (10)(40)$$

$$+ (20)(55) + (20)(65) = 3700$$

Yes, this approximation is less because the graph of  $R$  is increasing on the interval.

- (d)  $\int_0^b R(t) dt$  is the total amount of fuel in gallons consumed for the first  $b$  minutes.  
 $\frac{1}{b} \int_0^b R(t) dt$  is the average value of the rate of fuel consumption in gallons/min during the first  $b$  minutes.

2 : { 1 : a difference quotient using numbers from table and interval that contains 45  
 1 : 1.5 gal/min<sup>2</sup>

2 : { 1 :  $R''(45) = 0$   
 1 : reason

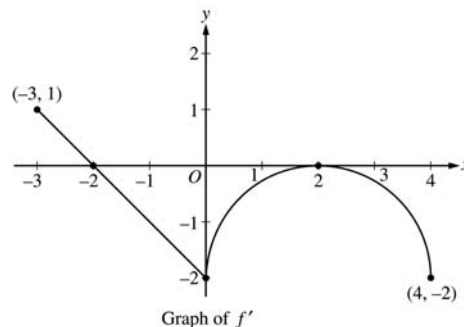
2 : { 1 : value of left Riemann sum  
 1 : "less" with reason

3 : { 2 : meanings  
 1 : meaning of  $\int_0^b R(t) dt$   
 1 : meaning of  $\frac{1}{b} \int_0^b R(t) dt$   
 < - 1 > if no reference to time  $b$   
 1 : units in both answers

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**Question 4**

Let  $f$  be a function defined on the closed interval  $-3 \leq x \leq 4$  with  $f(0) = 3$ . The graph of  $f'$ , the derivative of  $f$ , consists of one line segment and a semicircle, as shown above.



- (a) On what intervals, if any, is  $f$  increasing? Justify your answer.  
 (b) Find the  $x$ -coordinate of each point of inflection of the graph of  $f$  on the open interval  $-3 < x < 4$ . Justify your answer.  
 (c) Find an equation for the line tangent to the graph of  $f$  at the point  $(0, 3)$ .  
 (d) Find  $f(-3)$  and  $f(4)$ . Show the work that leads to your answers.

(a) The function  $f$  is increasing on  $[-3, -2]$  since  $f' > 0$  for  $-3 \leq x < -2$ .

2 :  $\left\{ \begin{array}{l} 1 : \text{interval} \\ 1 : \text{reason} \end{array} \right.$

(b)  $x = 0$  and  $x = 2$   
 $f'$  changes from decreasing to increasing at  $x = 0$  and from increasing to decreasing at  $x = 2$

2 :  $\left\{ \begin{array}{l} 1 : x = 0 \text{ and } x = 2 \text{ only} \\ 1 : \text{justification} \end{array} \right.$

(c)  $f'(0) = -2$   
 Tangent line is  $y = -2x + 3$ .

1 : equation

$$\begin{aligned} \text{(d)} \quad f(0) - f(-3) &= \int_{-3}^0 f'(t) dt \\ &= \frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = -\frac{3}{2} \end{aligned}$$

$$f(-3) = f(0) + \frac{3}{2} = \frac{9}{2}$$

$$\begin{aligned} f(4) - f(0) &= \int_0^4 f'(t) dt \\ &= -\left(8 - \frac{1}{2}(2)^2\pi\right) = -8 + 2\pi \end{aligned}$$

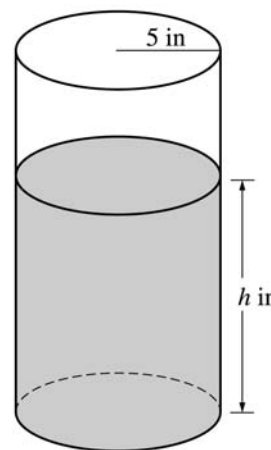
$$f(4) = f(0) - 8 + 2\pi = -5 + 2\pi$$

1 :  $\pm \left(\frac{1}{2} - 2\right)$   
 (difference of areas of triangles)  
 1 : answer for  $f(-3)$  using FTC  
 4 :  $\left\{ \begin{array}{l} 1 : \pm \left(8 - \frac{1}{2}(2)^2\pi\right)$   
 (area of rectangle - area of semicircle)  
 1 : answer for  $f(4)$  using FTC \end{array} \right.

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**Question 5**

A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let  $h$  be the depth of the coffee in the pot, measured in inches, where  $h$  is a function of time  $t$ , measured in seconds. The volume  $V$  of coffee in the pot is changing at the rate of  $-5\pi\sqrt{h}$  cubic inches per second. (The volume  $V$  of a cylinder with radius  $r$  and height  $h$  is  $V = \pi r^2 h$ .)



- (a) Show that  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$ .
- (b) Given that  $h = 17$  at time  $t = 0$ , solve the differential equation  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$  for  $h$  as a function of  $t$ .
- (c) At what time  $t$  is the coffeepot empty?

(a)  $V = 25\pi h$

$$\frac{dV}{dt} = 25\pi \frac{dh}{dt} = -5\pi\sqrt{h}$$

$$\frac{dh}{dt} = \frac{-5\pi\sqrt{h}}{25\pi} = -\frac{\sqrt{h}}{5}$$

(b)  $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$

$$\frac{1}{\sqrt{h}} dh = -\frac{1}{5} dt$$

$$2\sqrt{h} = -\frac{1}{5}t + C$$

$$2\sqrt{17} = 0 + C$$

$$h = \left(-\frac{1}{10}t + \sqrt{17}\right)^2$$

(c)  $\left(-\frac{1}{10}t + \sqrt{17}\right)^2 = 0$

$$t = 10\sqrt{17}$$

$$3 : \left\{ \begin{array}{l} 1 : \frac{dV}{dt} = -5\pi\sqrt{h} \\ 1 : \text{computes } \frac{dV}{dt} \\ 1 : \text{shows result} \end{array} \right.$$

$$5 : \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } h = 17 \\ \quad \text{when } t = 0 \\ 1 : \text{solves for } h \end{array} \right.$$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

1 : answer

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**Question 6**

Let  $f$  be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is  $f$  continuous at  $x = 3$ ? Explain why or why not.  
 (b) Find the average value of  $f(x)$  on the closed interval  $0 \leq x \leq 5$ .  
 (c) Suppose the function  $g$  is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$$

where  $k$  and  $m$  are constants. If  $g$  is differentiable at  $x = 3$ , what are the values of  $k$  and  $m$ ?

- (a)  $f$  is continuous at  $x = 3$  because

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = 2.$$

$$\text{Therefore, } \lim_{x \rightarrow 3} f(x) = 2 = f(3).$$

(b) 
$$\begin{aligned} \int_0^5 f(x) dx &= \int_0^3 f(x) dx + \int_3^5 f(x) dx \\ &= \frac{2}{3}(x+1)^{3/2} \Big|_0^3 + \left(5x - \frac{1}{2}x^2\right) \Big|_3^5 \\ &= \left(\frac{16}{3} - \frac{2}{3}\right) + \left(\frac{25}{2} - \frac{21}{2}\right) = \frac{20}{3} \end{aligned}$$

$$\text{Average value: } \frac{1}{5} \int_0^5 f(x) dx = \frac{4}{3}$$

- (c) Since  $g$  is continuous at  $x = 3$ ,  $2k = 3m + 2$ .

$$g'(x) = \begin{cases} \frac{k}{2\sqrt{x+1}} & \text{for } 0 < x < 3 \\ m & \text{for } 3 < x < 5 \end{cases}$$

$$\lim_{x \rightarrow 3^-} g'(x) = \frac{k}{4} \text{ and } \lim_{x \rightarrow 3^+} g'(x) = m$$

Since these two limits exist and  $g$  is differentiable at  $x = 3$ , the two limits are equal. Thus  $\frac{k}{4} = m$ .

$$8m = 3m + 2; m = \frac{2}{5} \text{ and } k = \frac{8}{5}$$

2 :  $\left\{ \begin{array}{l} 1 : \text{answers "yes" and equates the} \\ \text{values of the left- and right-hand} \\ \text{limits} \\ 1 : \text{explanation involving limits} \end{array} \right.$

4 :  $\left\{ \begin{array}{l} 1 : k \int_0^3 f(x) dx + k \int_3^5 f(x) dx \\ \text{(where } k \neq 0) \\ 1 : \text{antiderivative of } \sqrt{x+1} \\ 1 : \text{antiderivative of } 5-x \\ 1 : \text{evaluation and answer} \end{array} \right.$

3 :  $\left\{ \begin{array}{l} 1 : 2k = 3m + 2 \\ 1 : \frac{k}{4} = m \\ 1 : \text{values for } k \text{ and } m \end{array} \right.$