

## Particle Motion Cheat Sheet:

## 1. Position

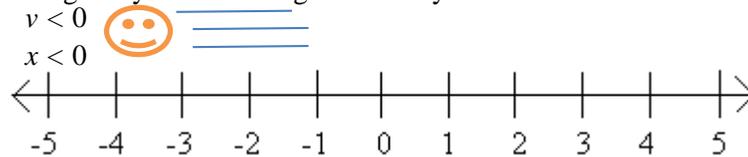
- Particles are moving on the  $x$  axis,  $x(t)$ , or may be disguised as real world problems.
- If  $p$  is the position of the particle at time  $t$ , then  $p(t) - p(a) = \int_a^t v(s) ds$ , where  $v$  is the velocity.

## 2. Velocity

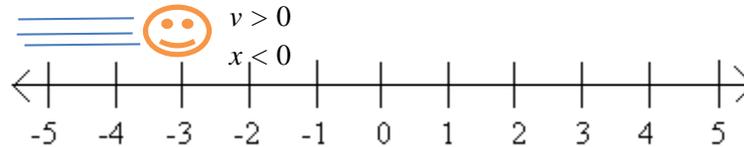
- A particle is moving to the left/backwards when  $v(t) < 0$ .
- A particle is moving to the right/forwards when  $v(t) > 0$ .
- Units: distance/time
- Derivative of position function:  $\frac{d}{dt} p(t)$
- Anti-derivative of acceleration function:  $v(t) - v(a) = \int_a^t a(s) ds$

## f. Movement with respect to the origin

- Moving away from the origin/ Velocity and Position are the same sign.

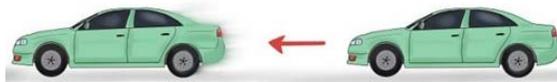


- Moving towards the origin/ Velocity and Position are different signs



## 3. Speed

- The absolute value of the velocity function:  $|v(t)|$
- Units: distance/time
- Increasing
  - the acceleration and the velocity are the same sign.
- Decreasing
  - the acceleration and the velocity are different sign.
- distance a particle travels from  $t = a$  to  $t = b$  is given by  $\int_a^b |v(t)| dt$  (the integral of speed)

-50 mph ( $t = 2$  sec)-20 mph ( $t = 1$  sec)

velocity and acceleration are negative, but car's speed is increasing.

## 4. Acceleration

- a. The second derivative of the position function,  $\frac{d^2 p}{dt^2} = a(t)$
- b. The derivative of velocity,  $\frac{dv}{dt} = a(t)$
- Changing Direction = Velocity changes sign
  - Farthest to the Left = Absolute minimum of position (check endpoints and local min values)
  - Farthest to the Right = Absolute maximum of position (check endpoints and local max values)
  - Particle at rest = Velocity is zero
  - Particle is moving fastest = Absolute maximum of speed (not velocity)

Function	Derivative	Integral
$x(t)$	N/A	$x(t) = x(a) + \int_a^t v(u) du$
$v(t)$	$x'(t)$	$v(t) = v(a) + \int_a^t a(u) du$
$a(t)$	$v'(t)$	N/A