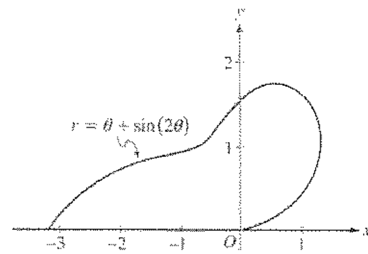
Question 2



The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

(a) $\text{Area} = \frac{1}{2}\int_0^{\pi} r^2 \, d\theta$

$= \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2 \, d\theta = 4.382$

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

(b) $-2 = r\cos(\theta) = (\theta + \sin(2\theta))\cos(\theta)$

$\theta = 2.786$

2 : { 1 : equation; 1 : answer }

(c) Since $\frac{dr}{d\theta} < 0$ for $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, r is decreasing on this interval. This means the curve is getting closer to the origin.

2 : { 1 : information about r; 1 : information about the curve }

(d) The only value in $\left[0, \frac{\pi}{2}\right]$ where $\frac{dr}{d\theta} = 0$ is $\theta = \frac{\pi}{3}$.

θ	r
0	0
$\frac{\pi}{3}$	1.913
$\frac{\pi}{2}$	1.571

The greatest distance occurs when $\theta = \frac{\pi}{3}$.

2 : { 1 : $\theta = \frac{\pi}{3}$ or 1.047; 1 : answer with justification }

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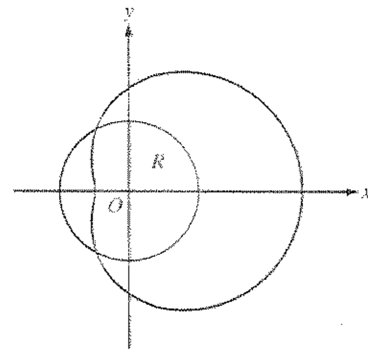
Question 3



The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

(a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

(b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

(a) Area $= \frac{2}{3}\pi(2)^2 + \frac{1}{2}\int_{2\pi/3}^{4\pi/3}(3 + 2\cos\theta)^2\,d\theta$
$= 10.370$
	4 : { 1 : area of circular sector
	2 : integral for section of limaçon
	  1 : integrand
	  1 : limits and constant
	1 : answer

(b) $\left.\frac{dr}{dt}\right\|_{\theta=\pi/3} = \left.\frac{dr}{d\theta}\right\|_{\theta=\pi/3} = -1.732$ The particle is moving closer to the origin, since $\frac{dr}{dt} < 0$ and $r > 0$ when $\theta = \frac{\pi}{3}$.	2 : { 1 : $\left.\frac{dr}{dt}\right\|_{\theta=\pi/3}$ 1 : interpretation
(c) $y = r\sin\theta = (3 + 2\cos\theta)\sin\theta$ $\left.\frac{dy}{dt}\right\|_{\theta=\pi/3} = \left.\frac{dy}{d\theta}\right\|_{\theta=\pi/3} = 0.5$ The particle is moving away from the x-axis, since $\frac{dy}{dt} > 0$ and $y > 0$ when $\theta = \frac{\pi}{3}$.	3 : { 1 : expression for y in terms of θ 1 : $\left.\frac{dy}{dt}\right\|_{\theta=\pi/3}$ 1: interpretation

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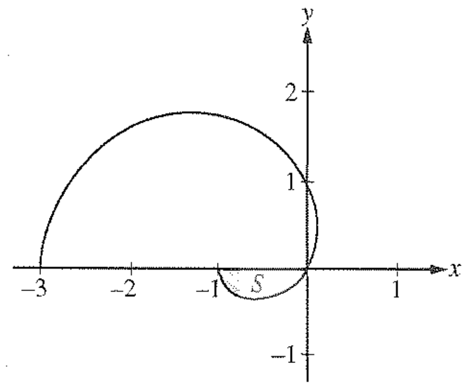
Question 4



The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

(a) Write an integral expression for the area of S.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.

(a) $r(0) = -1;\ r(\theta) = 0$ when $\theta = \frac{\pi}{3}$.

Area of $S = \frac{1}{2}\int_0^{\pi/3} (1 - 2\cos\theta)^2\, d\theta$

$2: \begin{cases} 1: \text{limits and constant} \\ 1: \text{integrand} \end{cases}$

(b) $x = r\cos\theta$ and $y = r\sin\theta$

$$\frac{dr}{d\theta} = 2\sin\theta$$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta = 4\sin\theta\cos\theta - \sin\theta$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta = 2\sin^2\theta + (1 - 2\cos\theta)\cos\theta$$

$4: \begin{cases} 1: \text{uses } x = r\cos\theta \text{ and } y = r\sin\theta \\ 1: \frac{dr}{d\theta} \\ 2: \text{answer} \end{cases}$

(c) When $\theta = \frac{\pi}{2}$, we have $x = 0,\ y = 1$.

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{2}} = \left.\frac{dy/d\theta}{dx/d\theta}\right|_{\theta=\frac{\pi}{2}} = -2$$

The tangent line is given by $y = 1 - 2x$.

$3: \begin{cases} 1: \text{values for } x \text{ and } y \\ 1: \text{expression for } \frac{dy}{dx} \\ 1: \text{tangent line equation} \end{cases}$

Question 2

The polar curve r is given by $r(\theta) = 3\theta + \sin\theta$, where $0 \le \theta \le 2\pi$.

(a) Find the area in the second quadrant enclosed by the coordinate axes and the graph of r.

(b) For $\frac{\pi}{2} \le \theta \le \pi$, there is one point P on the polar curve r with x-coordinate -3. Find the angle θ that corresponds to point P. Find the y-coordinate of point P. Show the work that leads to your answers.

(c) A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$ and such that $\frac{d\theta}{dt} = 2$. Find $\frac{dy}{dt}$ at the instant that $\theta = \frac{2\pi}{3}$, and interpret the meaning of your answer in the context of the problem.

Solution	Scoring
(a) Area $= \frac{1}{2}\int_{\pi/2}^{\pi}(r(\theta))^2\,d\theta = 47.513$	3 : { 1 : integrand 1 : limits and constant 1 : answer
(b) $-3 = r(\theta)\cos\theta = (3\theta + \sin\theta)\cos\theta$ $\theta = 2.01692$ $y = r(\theta)\sin(\theta) = 6.272$	3 : { 1 : equation 1 : value of θ 1 : y-coordinate
(c) $y = r(\theta)\sin\theta = (3\theta + \sin\theta)\sin\theta$ $\left.\frac{dy}{dt}\right\|_{\theta=2\pi/3} = \left[\frac{dy}{d\theta}\cdot\frac{d\theta}{dt}\right]_{\theta=2\pi/3} = -2.819$ The y-coordinate of the particle is decreasing at a rate of 2.819.	3 : { 1 : uses chain rule 1 : answer 1 : interpretation

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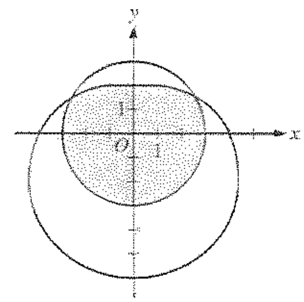
Question 2



The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.

(a) Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin\theta$. Find the area of S.

(b) A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \le t \le 2$ for which the x-coordinate of the particle's position is -1.

(c) For the particle described in part (b), find the position vector in terms of t. Find the velocity vector at time $t = 1.5$.

(a) Area $= 6\pi + \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4 - 2\sin\theta)^2\,d\theta = 24.709$ (or 24.708)	3 : { 1 : integrand 1 : limits and constant 1 : answer
(b) $x = r\cos\theta \Rightarrow x(\theta) = (4 - 2\sin\theta)\cos\theta$ $x(t) = \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right)$ $x(t) = -1$ when $t = 1.428$ (or 1.427)	3 : { 1 : $x(\theta)$ or $x(t)$ 1 : $x(\theta) = -1$ or $x(t) = -1$ 1 : answer
(c) $y = r\sin\theta \Rightarrow y(\theta) = (4 - 2\sin\theta)\sin\theta$ $y(t) = \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right)$ Position vector $= \langle x(t), y(t)\rangle$ $= \left\langle \left(4 - 2\sin\left(t^2\right)\right)\cos\left(t^2\right), \left(4 - 2\sin\left(t^2\right)\right)\sin\left(t^2\right)\right\rangle$ $v(1.5) = \langle x'(1.5), y'(1.5)\rangle$ $= \langle -8.072, -1.673\rangle$ (or $\langle -8.072, -1.672\rangle$)	3 : { 2 : position vector 1 : velocity vector