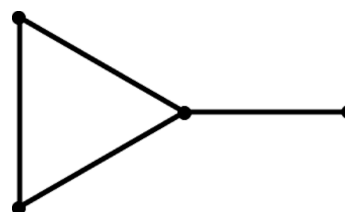


## *A Trio of Graph Puzzles*

Work on the puzzles below in any order you wish, either individually or with one or two other workshop participants. Our goal as a group is to find, present, and agree on answers to all three puzzles.

1. Suppose that we place five chips on the rightmost vertex of the graph shown here. We then perform a sequence of moves. On each move we identify all vertices having at least as many chips as neighbors. Every such vertex will distribute one chip to each neighbor, with all chips passing between vertices simultaneously. Without actually performing the sequence of moves, predict whether the process will continue indefinitely or eventually terminate. Then determine what actually occurs in the long run.



2. The vertices in the graph at right are colored red, blue and green to begin. We randomly select any vertex and next choose one of its neighbors, also at random. We then color the selected vertex the same color as its chosen neighbor. We repeat this until all vertices are the same color. What is the probability that all the vertices are now red?



3. Two players take turns removing either an edge, or a vertex along with all adjacent edges, beginning with a square graph. (Thus if the first player were to remove the upper right vertex then the top and right edges would also disappear.) Which player can guarantee to win the game, assuming they use the best possible strategy?

