Series Convergence Tests

1. An infinite **Geometric series**, $\sum_{n=0}^{\infty} ar^n$ converges to $\frac{a}{1-r}$ if and only if $|r|<1$.

2. **Divergence Test:** If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=k}^{\infty} a_n$ diverges.

3. A **Telescoping series** converges to a real number and the sum is found using partial fractions.

4. **Integral Test:** If $f(n) = a_n$, then $\sum_{n=1}^{\infty} a_n$ converges if and only $\int_1^{\infty} f(x)dx$ converges.

5. **Comparison Test:** If

- $a_n \le c_n$ for all $n \in \mathbb{N}$, and $\sum_{n=0}^{\infty} c_n$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.
- and $a_n \ge d_n$ for all $n \in \mathbb{N}$, and $\sum_{n=0}^{\infty} d_n$ diverges, then $\sum_{n=0}^{\infty} a_n$ diverges.

6. **Ratio Test:** If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$, then

- $L > 1$, then $\sum_{n=0}^{\infty} a_n$ diverges.
- $L < 1$, then $\sum_{n=0}^{\infty} a_n$ converges.
- $L = 1$, then more information is needed.

7. **Root Test:** If $\lim_{n\to\infty} \sqrt[n]{a_n} = L$, then

- $L > 1$, then $\sum_{n=0}^{\infty} a_n$ diverges.
- $L < 1$, then $\sum_{n=0}^{\infty} a_n$ converges.
- $L = 1$, then more information is needed.

8. **P-series Test:** If $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$.

9. **Limit Comparison Test:** If $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ with $a_n, b_n \ge 0$, for all n, and $c = \lim_{n\to\infty} \frac{a_n}{b_n}$ is finite then both series converge or both series diverge.

10. **Alternating Series Test:** If (a_n) is a positive sequence with $a_{n+1} \le a_n$, for all n and $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

AP Calculus BC Lesson 132 - Limit Comparison Test Name:

Example:

What series would you compare $\sum_{n=1}^{\infty} \frac{2n}{n^2+1}$ this to?

Take the limit of the ratio.

Do they both diverge or converge?

Practice Problems:

For each of the following, decide what to compare it to, take the limit of the ratio, then decide whether they converge or diverge.

A) $\sum_{n=1}^{\infty} \frac{3n+8}{5+4n^7}$

B) $\sum_{n=1}^{\infty} \frac{4\sqrt[3]{n}-2}{5n+1}$

C) $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n(n+3)^2}$

D) $\sum_{n=1}^{\infty} \frac{(n+1)^2 5^n}{n^2 \cdot 2^n}$

E) $\sum_{n=1}^{\infty} \frac{\sqrt[5]{n^{11}+1}}{\sqrt{n}}$

F) $\sum_{n=1}^{\infty} \frac{n+4}{n \cdot 4^n}$